Computer Graphics
Spring 2014 Final

NAME:

Prob #	1	2	3	4	5	6	7	8
Points	4	4	12	12	16	20	20	12

Time: 80 Minutes

NOTES:

a. **Credit is only given to the correct numerical values.**
b. **All numerical values must be calculated with three digits of accuracy after the decimal point.**
c. **Do not write on the back side of the papers.**

1. The RGB Value of a pixel is given as R=0.2 ; G=0.7 ; B=0.1
Find the CMYK values of this pixel:

C=1-R=1-0.2=0.8
M=1-G=1-0.7=0.3
Y=1-B=1-0.1=0.9
K=min(C,M,Y)=0.3

2. In OpenGL, all calls to "glVertex3d" should occur between which two other calls?

glBegin()
glEnd()

3. Given the triangle ABC in a three dimensional right-handed coordinate system,
A=(0,0,0), B=(10,0,0), C=(0,20,0)
Given the intensities at points A=1000 , B=2000 , and C=3000.

Find the intensity at point P(4,6,0) using Gouraud interpolative shading

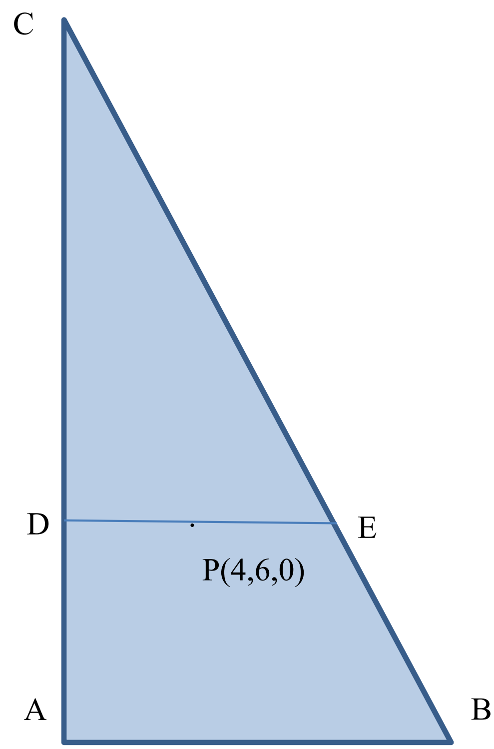
$$v_p = v_1 - (v_1 - v_2)\frac{(P_1 - P)}{(P_1 - P_2)}$$

$$I_D = 3000 - (3000 - 1000)\frac{(20-6)}{(20-0)} = 1600$$

$$I_E = 3000 - (3000 - 2000)\frac{(20-6)}{(20-0)} = 2300$$

$$x_E = 0 - (0 - 10)\frac{(20-6)}{(20-0)} = 7$$

$$I_p = 1600 - (1600 - 2300)\frac{(0-4)}{(0-7)} = 2000$$



4. Equation of a parametric surface is given as

$$x(u,v) = 400\,uv^2 - 200\,v^2 - 800\,v$$
$$y(u,v) = 300\,u^2v^2 - 600\,u$$
$$z(u,v) = 500\,u^2 + 700\,v - 200$$

Find the normal to this surface at point corresponding to u=0.5 and v=0.8

Normal to the surface @u=0.5 and v=0.8 is: ____________________________

$$\begin{cases} \frac{dx}{du} = 400v^2 \\ \frac{dy}{du} = 600uv^2 - 600 \\ \frac{dz}{du} = 1000u \end{cases} \quad \text{and} \quad \begin{cases} \frac{dx}{dv} = 800uv - 400v - 800 \\ \frac{dy}{dv} = 600u^2v \\ \frac{dz}{dv} = 700 \end{cases}$$

$$@u = 0.5, v = 0.8 \begin{cases} \frac{dx}{du} = 256 \\ \frac{dy}{du} = -408 \\ \frac{dz}{du} = 500 \end{cases} \quad \text{and} \quad @u = 0.5, v = 0.8 \begin{cases} \frac{dx}{dv} = -800 \\ \frac{dy}{dv} = 120 \\ \frac{dz}{dv} = 700 \end{cases}$$

Find cross product of the two vectors:

$N = (-345600\ ,\ -579200\ ,\ -295680)$

Or

$N = (345600\ ,\ 579200\ ,\ 295680)$

5. Consider a parametric cubic-linear surface.
$S(u,v)=[U]^T[M_B]^T[G][M_L][V]$

The geometry vector in the u direction is defined by Bezier and the geometry vector in the v direction is defined as p_0 , $\frac{dp_0}{dv}$
Find the geometry matrix [G] for this surface. Note: All elements should be specified explicitly as $p_{u,v}$ or derivatives of it. Do not use implicit forms such as p_1, p_2, p_3, p_4.

$$G_{BL} = \begin{bmatrix} p_{00} & \frac{dp_{00}}{dv} \\ p_{00} + \frac{1}{3}\frac{dp_{00}}{du} & \frac{dp_{00}}{dv} + \frac{1}{3}\frac{d^2 p_{00}}{dudv} \\ p_{10} - \frac{1}{3}\frac{dp_{10}}{du} & \frac{dp_{10}}{dv} - \frac{1}{3}\frac{d^2 p_{10}}{dudv} \\ p_{10} & \frac{dp_{10}}{dv} \end{bmatrix}$$

6. The viewing parameters for a parallel projection are given as:

VRP(WC)=(**0,0,0**) VPN(WC)=(**0, 0,1**)
VUP(WC)=(**0,1,0**) PRP (VRC)=(**10,20,50**)

u_{min} (VRC) = **-6** u_{max} (VRC) = **-2**
v_{min} (VRC) = **-2** v_{max} (VRC) = **6**
n_{min} (VRC) = **-4** n_{max} (VRC) = **1**

Find the sequence of transformations which will transform this viewing volume into a unit cube which is bounded by the planes: x=0 ; x=1 ; y=0 ; y=1 ; z=0 ; z=1

Matrix #2: Rx

1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1

Matrix #1: Translate

1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1

Matrix #4: Rz

1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1

Matrix #3: Ry

1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1

Matrix #6: Translate

1.000	0.000	0.000	6.000
0.000	1.000	0.000	2.000
0.000	0.000	1.000	4.000
0.000	0.000	0.000	1.000

Matrix #5: Shear

1.000	0.000	-0.280	0.000
0.000	1.000	-0.360	0.000
0.000	0.000	1.000	0.000
0.000	0.000	0.000	1.000

Matrix #7: Scale

0.250	0.000	0.000	0.000
0.000	0.125	0.000	0.000
0.000	0.000	0.200	0.000
0.000	0.000	0.000	1.000

7. Consider a bilinear surface S(u,v). The geometry vector for both u and v parameters are defined as p_0, p_1

Find the blending functions for this surface:

$$S(u,v) = \begin{bmatrix} u & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} v \\ 1 \end{bmatrix}$$

Multiply all the matrices and factor p_{00} ; p_{10} ; p_{01} ; p_{11}

Blending function for p_{00} is: $uv - u - v + 1$
Blending function for p_{10} is: $-uv + u$
Blending function for p_{01} is: $-uv + v$
Blending function for p_{11} is: uv

8. Given the triangle ABC in a three dimensional right-handed coordinate system, A=(4,0,0), B=(0,2,0), C=(0,0,2)
The light source with an intensity of I=10000 is located at (4,10,8) and the viewer (eye) is located at (0,6,6) and K_a=0 ; K_d=0.2 ; K_s=0.8 ; n=2

Given point P(0,1,1) on the triangle ABC:

a. Find the diffuse intensity at point P
Notes:
- Do not use any shading model
- Ignore fatt

Vector AB=

$\overrightarrow{AB} = (0,2,0) - (4,0,0) = (-4,2,0)$
$\overrightarrow{AC} = (0,0,2) - (4,0,0) = (-4,0,2)$
$\vec{N} = \overrightarrow{AB} X \overrightarrow{AC} = (4,8,8)$
$\hat{N} = (\ 0.3333 \quad 0.6667 \quad 0.6667)$
$\vec{L} = (4,10,8) - (0,1,1) = (4,9,7)$
$\hat{L} = (0.3310\ , \ 0.7448\ , \ 0.5793)$
$I_{Diffuse} = I * k_d * (\hat{N} \cdot \hat{L}) = 10000 * 0.2 * 0.9931 = 1986.25$
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b. Find the specular intensity at point P from the viewer's point of view
- Do not use any shading model
- Ignore fatt

$\vec{V} = (0,6,6) - (0,1,1) = (0,5,5)$
$\hat{V} = (0\ \ , 0.7071\ , 0.7071)$
$\hat{R} = 2 * \hat{N} * (\hat{N} \cdot \hat{L}) - \hat{L} = (0.3310\ , 0.5793, 0.7448)$
$\hat{R} \cdot \hat{V} = 0.9363$
$I_{Specular} = I * k_s * (\hat{R} \cdot \hat{V})^n = 7013.699$

$$R_z(\theta)=\begin{bmatrix}\cos\theta & -\sin\theta & 0 & 0\\ \sin\theta & \cos\theta & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\quad R_y(\theta)=\begin{bmatrix}\cos\theta & 0 & \sin\theta & 0\\ 0 & 1 & 0 & 0\\ -\sin\theta & 0 & \cos\theta & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\quad R_x(\theta)=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & \cos\theta & -\sin\theta & 0\\ 0 & \sin\theta & \cos\theta & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

$$M_{Hermite}=\begin{bmatrix}2 & -3 & 0 & 1\\ -2 & 3 & 0 & 0\\ 1 & -2 & 1 & 0\\ 1 & -1 & 0 & 0\end{bmatrix}\qquad M_{Bezier}=\begin{bmatrix}-1 & 3 & -3 & 1\\ 3 & -6 & 3 & 0\\ -3 & 3 & 0 & 0\\ 1 & 0 & 0 & 0\end{bmatrix}$$

How to convert a general parallel view volume into canonical perspective volume

Step 1: Translate VRP to origin
Step 2: Rotate VPN around x until it lies in the xz plane with positive z
Step 3: Rotate VPN around y until it aligns with the positive z axis.
Step 4: Rotate VUP around z until it lies in the yz plane with positive y
Step 5: Shear DOP such that it aligns with vpn
Step 6: Translate the lower corner of the view volume to the origin
Step 7: Scale such that the view volume becomes a unit cube

Calculation of R:

$$R = 2N(N \cdot L) - L$$